The Probability and Statistics Examination consists of 45 multiple-choice test questions. The test is a three-hour examination based on material usually covered in undergraduate mathematics courses in mathematical probability and statistics. This booklet provides examples of the types of test questions found in the Probability and Statistics Examination. Although the particular selection of topics covered in the examination may vary from year to year, the types and the general forms of questions will remain the same.

After covering the study material thoroughly, candidates may wish to test themselves on each set of questions by trying to answer correctly as many questions as possible in the allotted time of three hours. In so doing, candidates can identify those areas requiring further work and study as well as gain experience in answering multiple-choice test questions.

The Examination Committees do not intend to publish new sample examination questions each year. The committees intend, however, to update from time to time the sample questions included in this booklet.

The Probability and Statistics Examination is jointly administered by the Society of Actuaries and the Casualty Actuarial Society. It is jointly sponsored by: the American Academy of Actuaries, the Canadian Institute of Actuaries, the Casualty Actuarial Society, the Conference of Consulting Actuaries and the Society of Actuaries.

GENERAL INFORMATION

1. \( \binom{n}{r} \) is the number of combinations of \( n \) objects taken \( r \) at a time.

2. \( \ln(x) \) is the natural logarithm of \( x \).

3. If \( A \) and \( B \) are sets, then the union \( A \cup B \) of sets \( A \) and \( B \) is the set of all elements belonging either to set \( A \) or to set \( B \) or to both; the intersection \( A \cap B \) of sets \( A \) and \( B \) is the set of all elements belonging both to set \( A \) and to set \( B \); and the complement \( A' \) of set \( A \) is the set of all elements that do not belong to set \( A \).

4. \( \emptyset \) is the null or empty set.

5. All problems involving cards refer to an ordinary deck of 52 playing cards, consisting of 4 suits of 13 cards each. A bridge hand consists of 13 cards.

6. All problems involving dice refer to the ordinary six-sided die.

7. All problems involving a fair coin refer to a coin for which the probability of heads is \( \frac{1}{2} \) and the probability of tails is \( \frac{1}{2} \).

8. The mean of a random variable \( X \) is denoted by \( \mu_X = E(X) \); the variance of a random variable \( X \) is denoted by \( \sigma_X^2 = \text{Var}(X) \); the covariance of two random variables \( X \) and \( Y \) is denoted by \( \sigma_{XY} = \text{Cov}(X,Y) \); and the correlation coefficient of two random variables \( X \) and \( Y \) is denoted by \( \rho_{XY} = \text{Corr}(X,Y) \). The mean of a sample \( X_1, \ldots, X_n \) is denoted by \( \overline{X} = \frac{1}{n} \sum_{i=1}^{n} X_i \).
9. The table below gives the value of $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-w^2/2} dw$ for certain values of $x$. The integer of $x$ is given in the left column, and the first decimal place of $x$ is given in the top row. Since the density function of $x$ is symmetric, the value of the cumulative distribution function for negative $x$ can be obtained by subtracting from unity the value of the cumulative distribution for $x$.

![Normal distribution curve]

<table>
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<tr>
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<table>
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<td>2.58</td>
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2
10. The table below gives the value \( c \) for which \( P [\chi^2 < c] = p \) for a given number of degrees of freedom and a given value \( p \).

<table>
<thead>
<tr>
<th>df</th>
<th>Value of ( p )</th>
</tr>
</thead>
<tbody>
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<td>3</td>
<td>0.07 0.12 0.22 7.82 9.35 11.35 12.84</td>
</tr>
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<td>0.21 0.30 0.48 9.49 11.14 13.28 14.86</td>
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11. The table below gives the value $t$ for which $P[-\infty < T < t] = p$ for a given number of degrees of freedom and a given value of $p$.

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<th>Value of $p$</th>
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<th>Degrees of freedom for numerator</th>
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<td>7.19</td>
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<td>3.58</td>
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<td>0.995</td>
<td>6.87</td>
<td>6.54</td>
</tr>
</tbody>
</table>
1. A box contains 10 balls, of which 3 are red, 2 are yellow, and 5 are blue. Five balls are randomly selected with replacement. Calculate the probability that fewer than 2 of the selected balls are red.

   (A) 0.3601   (B) 0.5000   (C) 0.5282   (D) 0.8369   (E) 0.9167

2. Let \( X \) be a normal random variable with mean 0 and variance \( a > 0 \). Calculate \( P(X^2 < a) \).

   (A) 0.34   (B) 0.42   (C) 0.68   (D) 0.84   (E) 0.90

3. Workplace accidents are categorized in three groups: minor, moderate and severe. The probability that a given accident is minor is 0.5, that it is moderate is 0.4, and that it is severe is 0.1. Two accidents occur independently in one month. Calculate the probability that neither accident is severe and at most one is moderate.

   (A) 0.25   (B) 0.40   (C) 0.45   (D) 0.56   (E) 0.65

4. Let \( X_1, X_2, X_3 \) be uniform random variables on the interval \((0, 1)\) with \( \text{Cov}(X_i, X_j) = 1/24 \) for \( i, j = 1, 2, 3, i \neq j \). Calculate the variance of \( X_1 + 2X_2 - X_3 \).

   (A) 1/6   (B) 1/4   (C) 5/12   (D) 1/2   (E) 11/12

5. Let \( A, B, C \) and \( D \) be events such that \( B = A', C \cap D = \emptyset \), and \( P(A) = 1/4, P(B) = 3/4, P(C|A) = 1/2, P(C|B) = 1/8, P(D|A) = 1/4 \) and \( P(D|B) = 1/8 \). Calculate \( P(C \cup D) \).

   (A) 5/32   (B) 1/4   (C) 3/8   (D) 3/4   (E) 1

6. Let \( (X_1, Y_1), \ldots, (X_8, Y_8) \) be a random sample from a bivariate normal distribution with means \( \mu_X \) and \( \mu_Y \) and nonzero variances. The null hypothesis \( H_0: \mu_X = \mu_Y \) is rejected in favor of the alternative hypothesis \( H_1: \mu_X \neq \mu_Y \) if

\[
\frac{\sqrt{8|X - Y|}}{\sqrt{\frac{1}{8} \sum_{i=1}^{8} [(X_i - Y_i) - (X - Y)]^2}} > k.
\]

Determine the value of \( k \) for which the significance level (size) of the test is 0.05.

   (A) 1.64   (B) 1.90   (C) 1.96   (D) 2.31   (E) 2.37

7. A class contains 8 boys and 7 girls. The teacher selects 3 of the children at random and without replacement. Calculate the probability that the number of boys selected exceeds the number of girls selected.

   (A) \( \frac{512}{3375} \)   (B) \( \frac{89}{357} \)   (C) \( \frac{38}{135} \)   (D) \( \frac{1856}{3375} \)   (E) \( \frac{36}{85} \)

8. Let \( X \) and \( Y \) be discrete random variables with joint probability function \( p(x, y) \) given by the following table:

<table>
<thead>
<tr>
<th>( x )</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
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<td>0.05</td>
<td>0.15</td>
<td>0.05</td>
</tr>
<tr>
<td>( y = 1 )</td>
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<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>( y = 2 )</td>
<td>0.05</td>
<td>0.15</td>
<td>0.10</td>
<td>0.05</td>
</tr>
</tbody>
</table>

For this joint distribution \( E(X) = 2.85 \) and \( E(Y) = 1. \) Calculate \( \text{Cov}(X, Y) \).

   (A) -0.20   (B) -0.15   (C) 0.95   (D) 2.70   (E) 2.85
9. You are given the model \( E(Y_i) = \theta(x_i + x_i^2) \) and the following data:

<table>
<thead>
<tr>
<th></th>
<th>( x_i )</th>
<th>( y_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>14</td>
</tr>
</tbody>
</table>

Calculate the least squares estimate of \( \theta \).

(A) \( \frac{13}{92} \)  
(B) \( \frac{28}{23} \)  
(C) \( \frac{13}{10} \)  
(D) \( \frac{9}{7} \)  
(E) \( \frac{56}{5} \)

10. Let \( X_1, X_2 \) and \( X_3 \) be independent discrete random variables with probability functions

\[
P(X_i = k) = \begin{cases} \binom{n_i}{k} p^k (1-p)^{n_i-k} & k = 0, 1, \ldots, n_i \\ 0 & \text{otherwise} \end{cases}
\]

for \( i = 1, 2, 3 \) where \( 0 < p < 1 \). Determine the probability function of \( S = X_1 + X_2 + X_3 \), where positive.

(A) \( \binom{n_1 + n_2 + n_3}{s} p^s (1-p)^{n_1+n_2+n_3-s} \)

(B) \( \sum_{i=1}^{3} \frac{n_i}{n_1 + n_2 + n_3} \binom{n_i}{s} p^s (1-p)^{n_i-s} \)

(C) \( \prod_{i=1}^{3} \binom{n_i}{s} p^s (1-p)^{n_i-s} \)

(D) \( \sum_{i=1}^{3} \binom{n_i}{s} p^s (1-p)^{n_i-s} \)

(E) \( \binom{n_1 n_2 n_3}{s} p^s (1-p)^{n_1 n_2 n_3-s} \)

11. Let \( X \) be a random variable with distribution function

\[
F(x) = \begin{cases} 0 & \text{for } x < 0 \\ x/8 & \text{for } 0 \leq x < 1 \\ (2 + x)/8 & \text{for } 1 \leq x < 2 \\ (9 + x)/12 & \text{for } 2 \leq x < 3 \\ 1 & \text{for } x \geq 3 \end{cases}
\]

Calculate \( P(1 \leq X \leq 2) \).

(A) 1/8  
(B) 3/8  
(C) 7/16  
(D) 13/24  
(E) 19/24
12. Let \( X_1, \ldots, X_n \) be a random sample from a continuous distribution with density function

\[
f(x) = \begin{cases} 
\frac{\alpha^{2\alpha}}{x^{\alpha+1}} & \text{for } x \geq 2 \\
0 & \text{otherwise}
\end{cases}
\]

where \( \alpha > 0 \). Determine the maximum likelihood estimator of \( \alpha \).

(A) \( \min(X_1, \ldots, X_n) \)

(B) \( \frac{1}{n} \sum_{i=1}^{n} X_i \)

(C) \( \frac{n}{\sum_{i=1}^{n} \ln X_i} \)

(D) \( \max(X_1, \ldots, X_n) \)

(E) \( \frac{n}{\sum_{i=1}^{n} \ln X_i - n \ln 2} \)

13. Let \( X_1, \ldots, X_n \) be a random sample from a discrete distribution with probability function

\[
p(x) = \begin{cases} 
\theta(1 - \theta)^x & \text{for } x = 0, 1, 2, \ldots \\
0 & \text{otherwise}
\end{cases}
\]

where \( 0 < \theta < 1 \). Determine the form of the likelihood ratio test for rejecting the null hypothesis \( H_0 : \theta = 0.25 \) in favor of the alternative hypothesis \( H_1 : \theta \neq 0.25 \).

(A) \( \left( \frac{1+X}{4} \right)^n \left( \frac{3(1+X)}{4X} \right)^{\sum X_i} < k \)

(B) \( \left( \frac{1+X}{4} \right)^n \left( \frac{3(1+X)}{4X} \right)^{\sum X_i} < k \)

(C) \( \left( \frac{1+X}{2} \right)^n \left( \frac{1+X}{2X} \right)^{\sum X_i} < k \)

(D) \( \left( \frac{X}{4X} \right)^n \left( \frac{3}{4(1-X)} \right)^{\sum X_i} < k \)

(E) \( \left( \frac{1+X}{2} \right)^{n+\sum X_i} < k \)

14. Let \( X_1, \ldots, X_7 \) and \( Y_1, \ldots, Y_{14} \) be independent random samples from normal distributions with common mean \( \mu = 30 \) and common variance \( \sigma^2 > 0 \). The statistic \( W = \frac{2(Y-30)^2}{(X-30)^2} \) has an \( F \) distribution with \( c \) and \( d \) degrees of freedom. Determine \( c \) and \( d \).

(A) \( c = 1, d = 1 \)  \hspace{1cm} (B) \( c = 6, d = 13 \)  \hspace{1cm} (C) \( c = 7, d = 14 \)  \hspace{1cm} (D) \( c = 13, d = 6 \)  \hspace{1cm} (E) \( c = 14, d = 7 \)
15. Let \(X_1, \ldots, X_n\) be independent Poisson random variables with expectations \(\lambda_1, \ldots, \lambda_n\), respectively. Let \(Z = \sum_{i=1}^{n} a X_i\), where \(a\) is a constant. Determine the moment generating function of \(Z\).

(A) \(\exp \left( t \sum_{i=1}^{n} a \lambda_i + \frac{1}{2} t^2 \sum_{i=1}^{n} a^2 \lambda_i \right)\)

(B) \(\exp \left( \sum_{i=1}^{n} a \lambda_i (e^t - 1) \right)\)

(C) \(\exp \left( t \sum_{i=1}^{n} a \lambda_i + \frac{1}{2} t^2 \sum_{i=1}^{n} a^2 \lambda_i^2 \right)\)

(D) \(\exp \left( \sum_{i=1}^{n} \lambda_i (e^{at} - 1) \right)\)

(E) \(\left[ \prod_{i=1}^{n} \lambda_i \right] (e^{at} - 1)^n\)

16. Let \(X\) be a single observation from a continuous distribution with density function

\[ f(x) = \frac{1}{2} e^{-|x-\theta|} \text{ for } -\infty < x < \infty. \]

The null hypothesis \(H_0 : \theta = 0\) is tested against the alternative hypothesis \(H_1 : \theta = 1\). The null hypothesis is rejected if \(X > k\). The probability of a Type I error is 0.05. Calculate the probability of a Type II error.

(A) 0.0184  (B) 0.1359  (C) 0.8641  (D) 0.9500  (E) 0.9816

17. Let \(X\) and \(Y\) be continuous random variables with joint density function \(f(x, y)\), where \(f\) is nonzero for \(x \geq 1\) and \(y \geq 1\). Let \(Z = XY\) and \(W = X/Y\). Determine the joint density of \(Z\) and \(W\), where positive.

(A) \(\sqrt{f(zw, \frac{z}{w})}\)  (B) \(\frac{1}{zw} f(\sqrt{zw}, \sqrt{\frac{z}{w}})\)  (C) \(f(\sqrt{zw}, \sqrt{\frac{z}{w}})\)  (D) \(2w f(\sqrt{zw}, \sqrt{\frac{z}{w}})\)  (E) \(f(zw, \frac{z}{w})\)

18. Let \(X\) be a random variable with mean 0 and variance 4. Calculate the largest possible value of \(P(|X| \geq 8)\), according to Chebyshev’s inequality.

(A) 1/16  (B) 1/8  (C) 1/4  (D) 3/4  (E) 15/16

19. Let \(X_1, \ldots, X_n\) and \(Y_1, \ldots, Y_n\) be independent random samples from normal distributions with means \(\mu_X\) and \(\mu_Y\) and variances 2 and 4 respectively. The null hypothesis \(H_0 : \mu_X = \mu_Y\) is rejected in favor of the alternative hypothesis \(H_1 : \mu_X > \mu_Y\) if \(\bar{X} - \bar{Y} > \kappa\). Determine the smallest value of \(n\) for which a test of significance level (size) 0.025 has power of at least 0.5 when \(\mu_X = \mu_Y + 2\).

(A) 3  (B) 4  (C) 5  (D) 6  (E) 8

20. Five hypotheses are to be tested using five independent test statistics. A common significance level (size) for each test is desired which ensured that the probability of rejecting at least one hypothesis is 0.4, when all five hypotheses are true. Determine the desired common significance level (size).

(A) 0.040  (B) 0.080  (C) 0.097  (D) 0.167  (E) 0.400

21. Let \(X\) be a Poisson random variable with \(E(X) = \ln(2)\). Calculate \(E[\cos(\pi X)]\).

(A) 0  (B) 1/4  (C) 1/2  (D) 1  (E) 2 \ln(2)
22. Let $X$ and $Y$ be continuous random variables with joint density function
\[ f(x, y) = \begin{cases} 
1 & \text{for } 0 < x < 2 \text{ and } 0 < y < 1 - |x - 1| \\
0 & \text{otherwise}
\end{cases} \]
Calculate $\text{Var}(X)$.
(A) $\frac{1}{18}$  (B) $\frac{1}{6}$  (C) $\frac{1}{3}$  (D) 1  (E) $\frac{7}{6}$

23. Let $A$ and $B$ be events such that $P(A) = 0.7$ and $P(B) = 0.9$. Calculate the largest possible value of $P(A \cup B) - P(A \cap B)$.
(A) 0.20  (B) 0.34  (C) 0.40  (D) 0.60  (E) 1.60

24. Let $X$ and $Y$ be continuous random variables having a bivariate normal distribution with means $\mu_X$ and $\mu_Y$, common variance $\sigma^2$, and correlation coefficient $\rho_{XY}$. Let $F_X$ and $F_Y$ be the cumulative distribution functions of $X$ and $Y$, respectively. Determine which of the following is a necessary and sufficient condition for $F_X(t) \geq F_Y(t)$.
(A) $\mu_X \geq \mu_Y$  (B) $\mu_X \leq \mu_Y$  (C) $\mu_X \geq \rho_{XY} \mu_Y$  (D) $\mu_X \leq \rho_{XY} \mu_Y$  (E) $\rho_{XY} \geq 0$

25. The sum of the sample mean and median for ten distinct data points is equal to 20. The largest data is equal to 15. Calculate the sum of the sample mean and median if the largest data point were replaced by 25.
(A) 20  (B) 21  (C) 22  (D) 30  (E) 31

26. Let $X_1, \ldots, X_n$ be a random sample from a discrete distribution with probability function
\[ p(x) = \begin{cases} 
\theta & \text{for } x = 1 \\
\theta & \text{for } x = 2 \\
1 - 2\theta & \text{for } x = 3
\end{cases} \]
where $0 < \theta < 1/2$. Determine the method of moments estimator of $\theta$.
(A) $\frac{3 - X}{3}$  (B) $\frac{X + 1}{4}$  (C) $\frac{2X - 3}{6}$  (D) $\overline{X}$  (E) $\frac{1}{2} \overline{X}$

27. Let $X$ be a continuous random variable with density function
\[ f(x) = \begin{cases} 
\frac{2}{x^2} & \text{for } x \geq 2 \\
0 & \text{otherwise}
\end{cases} \]
Determine the density function of $Y = \frac{1}{X-\frac{1}{2}}$ for $0 < y \leq 1$.
(A) $\frac{1}{y^2}$  (B) $\frac{2}{(y+1)^2}$  (C) $\frac{2}{(y-1)^2}$  (D) $\frac{2y^2}{(y+1)^2}$  (E) $\frac{2(y+1)^2}{y^2}$

28. Let $X$ be the number of independent Bernouilli trials performed until a success occurs. Let $Y$ be the number of independent Bernouilli trials performed until 5 successes occur. A success occurs with probability $p$ and $\text{Var}(X) = 3/4$. Calculate $\text{Var}(Y)$.
(A) $\frac{3}{20}$  (B) $\frac{3}{4} \sqrt{p}$  (C) $\frac{3}{4}$  (D) $\frac{15}{4}$  (E) $\frac{75}{4}$

29. Let $X$ and $Y$ be discrete random variables with joint probability function
\[ p(x, y) = \begin{cases} 
\frac{2x+y+1}{9} & \text{for } x = 1, 2 \text{ and } y = 1, 2 \\
0 & \text{otherwise}
\end{cases} \]
Calculate $E(X/Y)$.
(A) $\frac{8}{9}$  (B) $\frac{5}{4}$  (C) $\frac{4}{3}$  (D) $\frac{25}{18}$  (E) $\frac{5}{3}$
30. Let \( X \) and \( Y \) be two independent random variables with moment generating functions
\[
M_X(t) = e^{t^2+2t} \\
M_Y(t) = e^{3t^2+t}
\]
Determine the moment generating function of \( X + 2Y \).
(A) \( e^{t^2+2t} + 2e^{3t^2+t} \) (B) \( e^{t^2+2t} + e^{12t^2+2t} \) (C) \( e^{7t^2+4t} \) (D) \( 2e^{4t^2+3t} \) (E) \( e^{13t^2+4t} \)

31. Let \( X \) and \( Y \) be independent random variables with \( \mu_X = 1, \mu_Y = -1, \sigma_X^2 = 1/2 \) and \( \sigma_Y^2 = 2 \). Calculate \( E \left[ (X + 1)^2 (Y - 1)^2 \right] \).
(A) 1 (B) 9/2 (C) 16 (D) 17 (E) 27

32. Let \( X \) be a single observation from a continuous distribution with density function
\[
f(x) = \begin{cases} 
\lambda e^{-\lambda x} & \text{for } x > 0 \\
0 & \text{otherwise}
\end{cases}
\]
where \( \lambda > 0 \). The null hypothesis \( H_0 : \lambda = 1 \) is tested against the alternative \( H_1 : \lambda > 1 \). Determine the critical region corresponding to the uniformly most powerful test of significance level (size) 0.05.
(A) \( X < 0.051 \) (B) \( X > 0.051 \) (C) \( X < 2.996 \) (D) \( X > 2.996 \) (E) \( X > 3.689 \)

33. Let \( X_1, \ldots, X_n \) be a random sample from a continuous distribution with density function
\[
f(x) = \begin{cases} 
\frac{e^{-x}}{1-e^{-\theta}} & \text{for } 0 < x < \theta \\
0 & \text{otherwise}
\end{cases}
\]
where \( 0 < \theta < \infty \). Determine the maximum likelihood estimator of \( \theta \).
(A) \( \left( \prod_{i=1}^{n} X_i \right)^{1/n} \) (B) \( \overline{X} \) (C) \( \ln \left( \frac{1}{1-e^{-\theta}} \right) \) (D) \( \min (X_1, \ldots, X_n) \) (E) \( \max (X_1, \ldots, X_n) \)

34. Let \( X \) and \( Y \) be continuous random variables with joint density function
\[
f(x, y) = \begin{cases} 
2 & \text{for } 0 < x < y < 1 \\
0 & \text{otherwise}
\end{cases}
\]
Determine the conditional density function of \( Y \) given \( X = x \), where \( 0 < x < 1 \).
(A) \( \frac{1}{y-x} \) for \( x < y < 1 \)
(B) \( 2(1 - x) \) for \( x < y < 1 \)
(C) \( 2 \) for \( x < y < 1 \)
(D) \( \frac{1}{y} \) for \( x < y < 1 \)
(E) \( \frac{1}{y} \) for \( x < y < 1 \)

35. Let \( X \) be a continuous random variable with density function
\[
f(x) = \begin{cases} 
6x(1-x) & \text{for } 0 < x < 1 \\
0 & \text{otherwise}
\end{cases}
\]
Calculate \( P \left[ \left| X - \frac{1}{2} \right| > \frac{1}{4} \right] \).
(A) 0.0521 (B) 0.1563 (C) 0.3125 (D) 0.5000 (E) 0.8000
36. Let $X$ and $Y$ be continuous random variables with joint cumulative distribution function

$$F(x, y) = \frac{1}{250} (20xy - x^2y - xy^2)$$

for $0 \leq x \leq 5$ and $0 \leq y \leq 5$. Determine $P(X > 2)$.

(A) $3/125$  (B) $11/50$  (C) $12/25$  (D) $\frac{3}{250} (39y - 3y^2)$  (E) $1 - \frac{1}{250} (36y - 2y^2)$

37. Let $X_1, \ldots, X_n$ be a random sample from a normal distribution with mean $\mu = 10$ and variance $\sigma^2 = 100$. Let $c \sum_{i=1}^{n} (X_i - 10)^2$ have a $\chi^2$ distribution with $d$ degrees of freedom. Determine $c$ and $d$.

(A) $c = 0.01$, $d = n - 1$

(B) $c = 0.01$, $d = n$

(C) $c = 0.1$, $d = n - 1$

(D) $c = 0.1$, $d = n$

(E) $c = (n - 1)^{-1/2}$, $d = n - 1$

38. A coin is tossed repeatedly and independently until the fourth head occurs. The null hypothesis $H_0 : \pi = 0.2$, where $\pi$ is the probability of heads, is rejected if the number of tosses required is less than or equal to 6. Determine the value of the power function of this test at $\pi = 0.3$.

(A) $\sum_{i=4}^{6} \binom{i}{3} (0.3)^4 (0.7)^{i-4}$

(B) $\sum_{i=4}^{6} \binom{i}{4} (0.2)^4 (0.8)^{i-4}$

(C) $\sum_{i=4}^{6} \binom{i}{4} (0.3)^4 (0.7)^{i-4}$

(D) $\sum_{i=4}^{6} \binom{i}{4} (0.3)^4 (0.7)^{i-4}$

(E) $\sum_{i=4}^{6} \binom{i}{3} (0.2)^4 (0.8)^{i-4}$

39. A bowl contains 10 balls, of which 4 are red and 6 are white. Balls are randomly selected with replacement from the bowl until 4 red balls have been selected. Let $X$ be the number of white balls drawn before the fourth red ball is selected. Calculate $P(X = 6)$.

(A) 0.0012  (B) 0.0187  (C) 0.0446  (D) 0.1003  (E) 0.2508

40. Let $X_1, \ldots, X_{100}$ be a random sample from a exponential distribution with mean $1/2$. Determine the approximative value of $P \left( \sum_{i=1}^{100} X_i > 57 \right)$ using the Central Limit Theorem.

(A) 0.08  (B) 0.16  (C) 0.31  (D) 0.38  (E) 0.46
41. Let $X$ be a continuous random variable with density function

$$f(x) = \begin{cases} \frac{1}{2} e^{-x/2} & \text{for } x > 0 \\ 0 & \text{otherwise} \end{cases}.$$

Determine the 25th percentile of the distribution of $X$.

(A) $\ln \left(\frac{4}{9}\right)$  
(B) $\ln \left(\frac{4}{16}\right)$  
(C) $\ln 4$  
(D) 2  
(E) $\ln 16$

42. Let $X$ be a continuous random variable with density function

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

for $-\infty < x < \infty$. Calculate $E(X|X > 0)$.

(A) 0  
(B) $1/\sqrt{2\pi}$  
(C) 1/2  
(D) $\sqrt{2/\pi}$  
(E) 1

43. Let $A$, $B$ and $C$ be independent events such that $P(A) = 0.5$, $P(B) = 0.6$ and $P(C) = 0.1$. Calculate $P(A' \cup B' \cup C)$.

(A) 0.69  
(B) 0.71  
(C) 0.73  
(D) 0.98  
(E) 1

44. The probability that a particular machine breaks down on any day is 0.2 and is independent of the breakdowns on any other day. The machine can break down only once per day. Calculate the probability that the machine breaks down two or more times in ten days.

(A) 0.0175  
(B) 0.0400  
(C) 0.2684  
(D) 0.6242  
(E) 0.9596

45. The weights of the animals in a population are normally distributed with variance 144. A random sample of 16 of the animals is taken. The mean weight of the sample is 200 pounds. Calculate the lower bound of the symmetric 90% confidence interval for the mean weight of the population.

(A) 140.96  
(B) 194.12  
(C) 194.75  
(D) 195.08  
(E) 198.77

**Keys**

| 01 à 05 | C | C | E | C | C |
| 06 à 10 | E | E | B | B | A |
| 11 à 15 | E | E | A | A | D |
| 16 à 20 | C | B | A | D | C |
| 21 à 25 | B | B | C | B | B |
| 26 à 30 | A | B | D | D | E |
| 31 à 35 | E | A | E | A | C |
| 36 à 40 | C | B | A | D | A |
| 41 à 45 | B | D | C | D | D |

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14 mai 2000